Lagrangian Approach to Totally Dissipative Evolutions in Wasserstein Spaces

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Aim of the talk is to study well-posedness for evolution equations/inclusions in the Wasserstein space $\mathcal{P}_2$ of probability measures driven by *totally dissipative* multivalued probability vector fields (MPVF). For this purpose, we lift the problem to an Hilbert space L^2 of random variables and we show that maximal totally dissipative MPVFs are in one-to-one correspondence with maximal dissipative operators in L^2 that are law invariant. This allows us to study the problem in an Hilbertian setting where the implicit Euler scheme can be applied to generate the unique Lagrangian flow for the differential inclusion of the corresponding Cauchy problem. This machinery is then imported in the Wasserstein framework where we obtain, as a byproduct, a Lagrangian characterization for the (unique) corresponding evolution of probability measures. This turns out to be also the unique *EVI* solution of the MPVF.

We also analyze the case of MPVFs satisfying only a (weaker, not total) dissipativity condition as in our previous work [1], as for e.g. the (opposite of) the subdifferential of convex functionals in $\mathcal{P}_2$. In this case, we provide sufficient conditions (related to a discrete approximation property) in order to obtain the well-posedness and the Lagrangian characterization results previously mentioned.

[1] Cavagnari G., Savaré G., Sodini G.E.: *Dissipative probability vector fields and generation of evolution semigroups in Wasserstein spaces*, Probab. Theory Relat. Fields (2022).

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